EFFECT OF RADIATION ON NATURAL CONVECTION FLOW FROM A POROUS VERTICAL PLATE IN PRESENCE OF HEAT GENERATION

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ABSTRACT

The effects of Radiation on Natural Convection Flow from a Porous Vertical Plate in Presence of Heat Generation have been presented here. The governing boundary layer equations are first transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations are then solved numerically using finite difference method together with Keller-Box scheme. The numerical results of the surface shear stress in terms of skin friction coefficient and the rate of heat transfer in terms of local Nusselt number, velocity as well as temperature profiles are shown graphically and tabular form for a selection of parameters set of consisting of heat generation parameter Q, radiation effect Rd, Prandtl number Pr.

Keywords: Radiation effect, Porous plate, Heat generation, Natural convection.

1. INTRODUCTION

The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution; consequently the particle deposition rate in nuclear reactors, electronic chips and semiconductor waters.

The effect of radiation on free convection has been drawn forth not only for its fundamental aspects but also for its significance in the contexts of space technology and processes involving high temperature. In the presence of heat generation, natural convection boundary layer flow from a porous vertical plate of a steady two dimensional viscous incompressible fluid and the radiated heat transfer has been investigated. In this analysis consideration had been given to grey gases that emit and absorb but do not scatter thermal radiation. Over the work it is assumed that the surface temperature of the porous vertical plate \( T_w \), is constant, where \( T_w > T_\infty \). Here \( T_\infty \) is the ambient temperature of the fluid, \( T \) is the temperature of the fluid in the boundary layer, \( g \) is the acceleration due to gravity, and the fluid is assumed to be a grey emitting and absorbing, but non scattering medium. In the present work following assumptions are made:

- Variations in fluid properties are limited only to those density variations which affect the buoyancy terms.
- The radiative heat flux in the x-direction is considered negligible in comparison with that in the y direction, where the physical coordinates (u, v) are velocity components along the (x, y) axes.

Vajravelu and Hadjinicolau[1] studied the heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation. In this study, they considered that the volumetric rate of heat generation \( q'' \) \([W/m^3]\) should be:
layer flow of the fluids along porous plate with radiation heat loss.

The present study deals with effects of radiation on natural convection flow from a porous vertical plate in presence of heat generation. The results will be obtained for different values of relevant physical parameters and will be shown in graphs as well as in tables.

The governing partial differential equations are reduced to locally non-similar partial differential forms by adopting some appropriate transformations. The transformed boundary layer equations are solved numerically using implicit finite difference scheme together with the Keller box technique[14] . Here, we have focused our attention on the evolution of the surface shear stress in terms of local skin friction and the rate of heat transfer in terms of local Nusselt number, velocity profiles as well as temperature profiles for selected values of parameters consisting of heat generation parameter Q, Prandtl number Pr and the radiation parameter R_d. In order to check the accuracy of our numerical results the present results are compared with[13].

2. FORMULATION OF THE PROBLEM

We have investigated the effects of radiation on natural convection flow from a porous plate in presence of heat generation. The fluid is assumed to be a grey, emitting and absorbing but non scattering medium. Over the work it is assumed that the surface temperature of the porous vertical plate, T_w, is constant, where T_w> T. The physical configuration considered is as shown in Fig.1.

The conservation equations for the flow characterized with steady, laminar and two dimensional boundary layers; under the usual Boussinesq approximation, the continuity, momentum and energy equations can be written as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
\rho(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}) = \mu \frac{\partial^2 u}{\partial y^2} + \rho g \beta (T - T_w) - \sigma_0 \beta_0^2 u \tag{2}
\]

\[
\rho c_p (u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}) = k \frac{\partial^2 T}{\partial y^2} + \frac{\partial q_{\text{rad}}}{\partial y} \tag{3}
\]
With the boundary conditions

\[ x = 0, y > 0, u = 0, T = T_{\infty}, \]
\[ y = 0, x > 0, u = 0, v = -V, T = T_{w}, \]
\[ y \to \infty, x > 0, u = 0, T = T_{\infty} \]  \hspace{1cm} (4)

where \( \rho \) is the density, \( \beta_0 \) is the strength of magnetic field, \( \sigma_0 \) is the electrical conduction, \( k \) is the thermal conductivity, \( \beta \) is the coefficient of thermal expansion, \( \nu \) is the reference kinematic viscosity \( \nu = \mu/\rho \), \( \mu \) is the viscosity of the fluid, \( C_p \) is the specific heat due to constant pressure and \( q_r \) is the radiative heat flux in the \( y \) direction.

\[ \eta = V_y / \nu^2, \xi = V / \nu^2 g \beta \Delta T, \]
\[ \psi = V^{-3} \nu^2 g \beta \Delta T \xi^3 \left[ f + \xi^2 \right] / 4, \]
\[ \theta = T - T_{\infty} / T_{w} - T_{\infty}, \]
\[ R_d = 4 \sigma T_r^3 / k(a + \sigma_s) \]

Where, \( \theta \) is the non-dimensional temperature function, \( \theta_s \) is the surface temperature parameter and \( R_d \) is the radiation parameter.

Substituting (6) into Equations (1, 2, 3) leads to the following non-dimensional equations

\[ f'' + \theta - 2 f' = 2 f' + 2 \xi f', \]
\[ \frac{1}{Pr} \frac{\partial}{\partial \eta} \left[ 1 + \frac{4}{3} Rd \left( 1 + (\theta - 1) \right)^3 \right] \frac{\partial \theta}{\partial \eta} + 3 f' + \xi \theta' \]
\[ = \xi \left( f \frac{\partial f}{\partial \xi} \frac{\partial \xi}{\partial \theta} \right) \]  \hspace{1cm} (8)

Where \( Pr = \nu C_p / k \) is the Prandtl number is the heat generation parameter and \( M = \beta_0 \sigma_0 / \nu \rho \) is the magneto hydrodynamic parameter.

The boundary conditions (4) become

\[ f = 0, f' = 0, \theta = 1 \text{ at } \eta = 0 \]
\[ f' = 0, \theta = 0 \text{ as } \eta \to \infty \]  \hspace{1cm} (9)

The solution of equations (6), (8) enable us to calculate the non dimensional velocity components \( u, v \) from the following expressions

\[ \bar{u} = \frac{\nu^2}{V g \beta \left( T_w - T_{\infty} \right)} u = \xi^2 f' \left( \xi, \eta \right), \]
\[ \bar{v} = \frac{v}{\nu} = \xi^{-1} \left( 3 f + \xi - \eta f' + \xi \frac{\partial f}{\partial \xi} \right) \]  \hspace{1cm} (10)

In practical applications, the physical quantities of principle interest are the shearing stress \( \tau_s \) and the rate of heat transfer in terms of the skin-friction coefficients \( C_f \) and Nusselt number \( Nu_s \) respectively, which can be written as

\[ Nu_s = \frac{V}{\nu \Delta T} (q_c + q_r) \eta = 0, C_f = \frac{V}{g \beta \Delta T} (\tau) \eta = 0 \]  \hspace{1cm} (11)

Figure 1. The coordinate system and the physical model.

In order to reduce the complexity of the problem and to provide a means of comparison with future studies that will employ a more detail representation for the radiative heat flux; we will consider the optically thick radiation limit. Thus we will approximate the radiative heat flux term of principle interest are the shearing stress \( \tau_s \) and the rate of heat transfer in terms of the skin-friction coefficients \( C_f \) and Nusselt number \( Nu_s \) respectively, which can be written as

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\[ Nu_s = \frac{V}{\nu \Delta T} (q_c + q_r) \eta = 0, C_f = \frac{V}{g \beta \Delta T} (\tau) \eta = 0 \]  \hspace{1cm} (11)
where  \( \tau_w = \mu \left( \frac{\partial u}{\partial \eta} \right)_{\eta=0} \) and

\[
q_e = -k \left( \frac{\partial T}{\partial \eta} \right)_{\eta=0}
\]  

(12)

\( q_e \) is the conduction heat flux.

Using the Equations (6) and the boundary condition (9) into (11 and 12), we get

\[
C_f(x) = \xi f''(x,0)
\]

\[
Nu_x = -\xi^{-1} \left( 1 + \frac{4}{3} Rd \theta_w^2 \right) \theta'(x,0)
\]

(13)

The values of the velocity and temperature distribution are calculated respectively from the following relations:

\[
\bar{u} = \xi^2 f' (\xi \eta), \quad \theta = \theta(x,y)
\]

3. METHOD OF SOLUTION

Solutions of the local non similar partial differential equation (7) to (8) subjected to the boundary condition (9) are obtained by using implicit finite difference method with Keller-Box Scheme[14], which has been described in details by Cebeci[15].

4. RESULTS AND DISCUSSION

In this exerction the effects of radiation on natural convection flow on a porous vertical plate in presence of heat generation is investigated. Numerical values of local rate of heat transfer are calculated in terms of Nusselt number \( Nu_x \) for the surface of the porous vertical plate from lower stagnation point to upper stagnation point, for different values of the aforementioned parameters and these are shown in tabular form in Table:1 and Table:2 and graphically in Figure 6-9. The effect for different values of heat generation parameter \( Q \) on local skin friction coefficient \( C_f \) and the local Nusselt number \( Nu_x \), as well as velocity and temperature profiles are displayed in Fig.2 and 6. The aim of these figures are to display how the profiles vary in \( \xi \), the selected streamwise coordinate.

Figure 2. (a) Velocity and (b) temperature profiles for different values of heat generation parameter \( Q \) with others fixed parameters.

Figures 2(a)-2(b) display results for the velocity and temperature profiles, for different values of heat generation parameter \( Q \) with Prandtl number \( Pr = 1.0 \), radiation parameter \( R_d = 0.1 \) and surface temperature parameter \( \theta_i = 1.1 \). It has been seen from Figures 2(a) and 2(b) that as the heat generation parameter \( Q \) increases, the velocity and the temperature profiles increase. The changes of velocity profiles in the \( \eta \) direction reveals the typical velocity profile for natural convection boundary layer flow, i.e., the velocity is zero at the boundary wall then the velocity increases to the peak value as \( \eta \) increases and finally the velocity approaches to zero (the asymptotic value). The maximum values of velocity are recorded to be 0.22590, 0.28724, 0.36866 and 0.46717 for \( Q = 0.0, 5.0, 10.0, 15.0 \) respectively which occur at the same point \( \eta = 0.83530 \) and for \( Q = 17.9 \), the maximum values of velocity are recorded to be 0.53057. Here, it is observed that at \( \eta = 0.97931 \),

\[ \cdots \]
the velocity increases by 106.8% as the heat generation parameter $Q$ changes from 0.0 to 15.0. The changes of temperature profiles in the $\eta$ direction also shows the typical temperature profile for natural convection boundary layer flow that is if the value of temperature profile is 1.0 (one) at the boundary wall then the temperature profile decreases gradually along $\eta$ direction for the value $Q$ less than 1.0 to the asymptotic value. But for $Q \geq 1.0$ the temperature profile increases (at $\eta = 0.68459$ temperature is 2.20416 for $Q = 17.9$) and again it decreases gradually along $\eta$ direction to the asymptotic value.

Here, as the radiation parameter $R_d$ increases, the velocity profile increases and the temperature profile increases slightly such that there exists a local maximum of the velocity within the boundary layer, but velocity increases near the surface of the vertical porous plate and then temperature decreases and finally approaches to zero.

Figure 3. (a) Velocity and (b) temperature profiles for different values of radiation parameter $R_d$ with others fixed parameters.

Figure 4. (a) Velocity and (b) temperature profiles for different values of heat flux parameter $\theta_w$ with others fixed parameters.

The effect for different values of radiation parameter $R_d$ the velocity and temperature profiles in case of Prandtl number $Pr = 1.0$, heat generation parameter $Q = 2.0$ and surface temperature parameter $\theta_w = 1.1$ are shown in Figures 3(a)-3(b).
The effect of different values of surface temperature parameter $\theta_w$, the velocity and temperature profiles while Prandtl number $Pr = 1.0$, heat generation parameter $Q = 2.0$ and radiation parameter $R_d = 0.1$ are shown in Figures 4(a)-4(b). Here, as surface temperature parameter $\theta_w$ increases, the velocity profile increases and the temperature profile increases such that there exists a local maximum of the velocity within the boundary layer, but velocity increases near the surface of the vertical porous plate and then temperature decreases and finally approaches to zero.

However, in Figures 5(a)-5(b), it is shown that when the Prandtl number $Pr$ increases with $\theta_w = 1.1$, $R_d = 0.1$ and $Q = 2.0$, both the velocity and temperature profiles decrease.

Figures 5(a) and 5(b) show that as the Prandtl number increases, the velocity and temperature profiles increase. This is because the Prandtl number is a measure of the ratio of momentum to thermal diffusion, and an increase in Prandtl number indicates a decrease in thermal diffusion relative to momentum diffusion. Consequently, the velocity profile increases, and the temperature profile decreases. This is evident in the graphs where the velocity profiles rise and the temperature profiles decrease as the Prandtl number increases.

The effect of different values of radiation parameter $R_d$ on the skin friction coefficient and the local rate of heat transfer while Prandtl number $Pr = 1.0$, heat generation parameter $Q = 2.0$ and surface temperature parameter $\theta_w = 1.1$ are shown in Figures 6(a) and 6(b). Here, it is observed that at $\xi = 0.23$, the skin friction increases by 18.52% and Nusselt number $Nu$ decreases by 97.336% as the heat generation parameter $Q$ changes from 17.9 to 0.0.

Figure 5. (a) Velocity and (b) temperature profiles for different values of prandtl number $Pr$ with other fixed parameters.

Figure 6. (a) Skin friction and (b) rate of heat transfer for different values of heat generation parameter $Q$ with other fixed parameters.
in the figures 7(a)-7(b). Here, as the radiation parameter \( R_d \) increases, the skin friction coefficient and heat transfer coefficient increase.

![Figure 7](image)

Figure 7. (a) Skin friction and (b) rate of heat transfer for different values of radiation parameter \( R_d \) with others fixed parameters

From Figures 8(a)-8(b), it can also easily be seen that an increase in the surface temperature parameter \( \theta_w \) leads to increase in the local skin friction coefficient \( C_{fx} \) and the local rate of heat transfer \( \dot{N}_u \), while Prandtl number \( Pr \) = 1.0, heat generation parameter \( Q = 2.0 \) and radiation parameter \( R_d = 0.1 \). It is also observed that at any position of \( \xi \), the skin friction coefficient \( C_{fx} \) increases and the local Nusselt number \( Nu \) increase as \( \theta_w \) increases from 0.0 to 3.2. This phenomenon can easily be understood from the fact that when the surface temperature parameter \( \theta_w \) increases, the temperature of the fluid rises and the thickness of the velocity boundary layer grow, i.e., the thermal boundary layer become thinner than the velocity boundary layer.

![Figure 8](image)

Figure 8. (a) Skin friction and (b) rate of heat transfer for different values of heat flux parameter \( \theta_u \) with others fixed parameters

The variation of the local skin friction coefficient \( C_{fx} \) and local rate of heat transfer \( \dot{N}_u \) for different values of Prandtl number \( Pr \) for \( \theta_w = 1.1 \), \( Rd = 0.1 \) and \( Q = 2.0 \) are shown in Figures 9(a)-9(b). We can observe from these figures that as the Prandtl number \( Pr \) increases, the skin friction coefficient decreases and rate of heat transfer increase.
Figure 9. (a) Skin friction and (b) rate of heat transfer for different values of Prandtl number parameter Pr with others fixed parameters.

Numerical results of skin friction and rate of heat transfer are calculated from equation (13) for the surface of the porous plate from lower stagnation point to upper stagnation point at \( \xi = 0.01 \) to \( \xi = 0.23 \). Numerical values of \( C_f \) and \( N_u \) are depicted in Table 1.

Here in the below table the values of skin friction coefficient \( C_f \) and Nusselt number \( N_u \) are recorded to be 0.18218, 0.17690, 0.16072, 0.15370 and 0.06579, 0.06612, 2.46974, 3.24161 for \( Q = 17.9, 15.0, 10.0, 0.50 \) and 0.00 respectively which occur at the same point \( \xi = 0.23 \). Here, it is observed that at \( \xi = 0.23 \), the skin friction increases by 18.52% and Nusselt number \( N_u \) decreases by 97.336% as the heat generation parameter \( Q \) changes from 17.9 to 0.0.

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5. COMPARISON OF THE RESULTS

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In order to verify the accuracy of the present work, the values of Nusselt number and skin friction for $Q = 0, R_d = 0.05, Pr = 1.0$ and various surface temperature $\theta_w = 1.1, \theta_w = 2.5$ at different position of $\xi$ are compared with Hossain et al. [13] as presented in Table 2. The results are found to be in excellent agreement.

6. CONCLUSION

The effect of radiation on natural convection flow on a porous vertical plate in presence of heat generation has been investigated for different values of relevant physical parameters including Prandtl number $Pr$, and surface temperature parameter $\theta_w$.

Significant effects of heat generation parameter $Q$ on velocity and temperature profiles as well as on skin friction and the rate of heat transfer have been found in this investigation but the effect of heat generation parameter $Q$ on rate of heat transfer is more significant. An increase in the values of heat generation parameter $Q$ leads to increase both the velocity and the temperature profiles, the local skin friction coefficient $C_{fx}$ increases at different position of $\eta$ and the local rate of heat transfer $Nu_x$ decreases at different position of $\xi$ for $\xi < 0.1$ and decrease asymptotically when $Pr=1.0$.

The increase in the values of radiation parameter $R_d$ leads to increase in the velocity profile, the temperature profile, the local skin friction coefficient $C_{fx}$ and the local rate of heat transfer $Nu_x$.

All the velocity profile, temperature profile, the local skin friction coefficient $C_{fx}$ and the local rate of heat transfer $Nu_x$ increases significantly when the values of surface temperature parameter $\theta_w$ increase.

The increase in Prandtl number $Pr$ leads to decrease in all the velocity profile, the temperature profile, the local skin friction coefficient $C_{fx}$ but the local rate of heat transfer $Nu_x$ increase.

Nomenclatures

- $f$: Dimensionless stream function
- $g$: Acceleration due to gravity
- $k$: Thermal conductivity
- $Nu_x$: Local Nusselt number
- $Pr$: Prandtl number
- $Q$: Heat generation parameter
- $q_w$: Heat flux at the surface
- $q_c$: Conduction heat flux
- $q_r$: Radiation heat flux
- $R_d$: Radiation parameter
- $T$: Temperature of the fluid in the boundary layer
- $T_x$: Temperature of the ambient fluid
- $T_w$: Temperature at the surface
- $(u, v)$: Dimensionless velocity components along the $(x, y)$ axes
- $V$: Wall suction velocity
- $(x, y)$: Axis in the direction along and normal to the surface respectively

Greek symbols

- $\alpha$: Equal to $\frac{4}{3} R_d$
- $\beta$: Coefficient of thermal expansion
- $\Delta$: Equal to $\theta_w - 1$
- $\Delta T$: Equal to $T_w - T_\infty$
- $\eta$: Similarity variable
- $\theta$: Dimensionless temperature function
- $\theta_w$: Surface temperature parameter
- $\mu$: Viscosity of the fluid
- $\nu$: Kinematic viscosity
- $\xi$: Similarity variable
- $\rho$: Density of the fluid
- $\sigma$: Stephman-Boltzman constant
- $\sigma_s$: Scattering co-efficient
- $\mu_f$: Absolute viscosity at the film temperature
- $\tau$: Coefficient of skin friction
- $\tau_w$: Shearing stress
- $\psi$: Non-dimensional stream function

Subscripts

- $w$: wall conditions
- $\infty$: Ambient temperature
References


