

AN ECONOMIC RELIABILITY TEST PLAN FOR GENERALIZED LOG-LOGISTIC DISTRIBUTION

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ABSTRACT

The generalized log-logistic distribution is considered as a probability model for the lifetime of the product. Sampling plans in which items that are put to test, to collect the life of the items in order to decide upon accepting or rejecting a submitted lot, are called reliability test plans. A test plan to determine the termination time of the experiment for a given sample size, producer's risk and termination number is constructed. The comparison of the present test plan over similar plans exists in the literature is established with respect to time of the experiment. Results are illustrated by an example.

Keywords: Generalized log-logistic distribution, reliability test plan, experimental time, producer's risk.

1. INTRODCUTION

The variable sampling plans are developed by proposing a decision rule to accept or reject a submitted lot of products on the basis of inspected measurable quality characteristic for sample products taken from the lot. As required by the principles of statistical inference, it is necessary to specify the probability distribution of variable characteristic. In the absence of such specification, it is taken as the well known normal distribution. However, if normal distribution is not a good fit to the data under consideration, the decision process constructed on this basis would be misleading. At the same time appeal to central limit theorem as a justification to normality assumption is not always valid as the sample size in quality control data is not large enough to adopt normality. In this backdrop, (Epstein, 1954), (Sobel & Tischendorf, 1959), (Goode & Kao, 1961), (Gupta & Groll, 1961), (Gupta, S.S., 1962), (Fertig & Mann, 1980), (Kantam & Rosaiah, 1998), (Kantam, Rosaiah, & Rao, 2001), (Baklizi, 2003), (Tsai & Wu, 2006), (Balakrishnan, Leiva, & Lopez, 2007), (Aslam & Shahbaz, 2007), (Aslam & Kantam, 2008) and (Rao, Ghitany, & Kantam, 2008) developed variety of distributions. Sampling plans in a new approach for log-logistic distribution are suggested by (Kantam, Rao, & Sriram, 2006), (Rosaiah, Kantam,

& Santtosh, 2007) for exponentiated log-logistic distribution, and (Rao, G S; Ghitany, M E; Kantam, R R, 2009) for Marshall-Olkin extended Lomax distribution. Our interest in this paper is working of a variable sampling plan parallel to the construction of a theoretical parametric test of hypothesis. Of these, the present paper deals with the construction of sampling plan with a new approach and its comparison with similar existing plans are given in Section 2. The operating characteristic is presented in Section 3. The results are illustrated by an example towards the end of Section 3.

In scaled densities, a null hypothesis about scale parameter such as "the scale parameter is greater than or equal to a specified value" is equivalent to saying that the 'average life of a product governed by the given scaled density exceeds a specified average life'. Acceptance of this hypothesis by a test procedure means that the sample life times used for testing indicate that the lot from which the sample is drawn is a good lot. Similarly rejection of the hypothesis implies that the lot is a bad lot. In this paper, we discussed the parallel between the testing of hypothesis in scaled densities and sampling plans.

2. THE SAMPLING PLAN

We know that the cumulative distribution function (c.d.f.) of the log-logistic distribution is

$$F(x) = \frac{(x/\sigma)^\beta}{[1+(x/\sigma)^\beta]}; x > 0, \sigma > 0, \beta > 1 \quad (1.1)$$

Define a new distribution on lines of (Kotz & Nadarajah, 2000) and (Nadarajah & Kotz, 2003), the c. d. f. is as follows:

$$G(x; \alpha, \beta, \sigma) = 1 - \left[1 - \frac{(x/\sigma)^\beta}{[1+(x/\sigma)^\beta]} \right]^\alpha$$

$$= 1 - [1+(x/\sigma)^\beta]^{-\alpha}; x > 0, \sigma > 0, \alpha > 0, \beta > 1 \quad (1.2)$$

It can be seen that this is the failure probability function of series system of α components, where lifetimes of the components are independently and identically distributed with common model as the log-logistic distribution given by (1.1).

We call (1.2) as the Generalized log-logistic distribution (GLLD). The corresponding probability density function (p.d.f.) is given by

$$g(x; \alpha, \beta, \sigma) = \frac{\alpha\beta}{\sigma} \frac{(x/\sigma)^{\beta-1}}{[1+(x/\sigma)^\beta]^{\alpha+1}}; x > 0, \sigma > 0, \alpha > 0, \beta > 1 \quad (1.3)$$

where σ is the scale parameter, β is shape parameter and α is an integer parameter representing the number of parallel components in the system.

Consider a null hypothesis “ $H_0 : \sigma > \sigma_0$ ”. If generalized log-logistic distribution is assumed as the model of a variable representing lifetimes of some items that have life and eventual failure, the above hypothesis is regarding the average life of those items in the population. If the H_0 is accepted on the basis of some sample lifetimes collected through a life testing experiment from out of a submitted lot of such items using any admissible statistical test procedure, we may conclude that the submitted lot has a better average life than what is specified. Accordingly, the lot that can be termed as a good lot and can be accepted. (Rao, Kantam, Rosaiah, & Prasad, 2012) constructed the minimum sample size required to make a decision about the lot, given the waiting time in terms of σ_0 (i.e., t/σ_0) and acceptance number c , some risk probability, say α^* . For a ready reference brief contents of (Rao, Kantam, Rosaiah, & Prasad, 2012) are given in the following lines. With a specified σ_0 of σ , the probability of detecting c or less failures (probability of accepting the lot) in a sample of size n is given by

$$\sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i} \quad (2.3)$$

where $p = G(t; \alpha, \beta, \sigma_0)$.

For $\sigma > \sigma_0$, the above probability of acceptance should increase. Therefore, if α^* is a prefixed risk probability this means

$$\sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i} \geq 1 - \alpha^* \quad (2.4)$$

For a given σ_0 and hence of t/σ_0 , this is a single inequality in two unknowns n and c assuming that the parameter α, β are known. Because, c is always less than n , inequality (2.4) can be solved for n with successive values of c from zero onwards. The earliest value of n that satisfies the inequality (2.4) are given for $\alpha = 2, \beta = 2, 1 - \alpha^* = 0.75, 0.90, 0.95, 0.99$ and $t/\sigma_0 = 0.315, 0.472, 0.629, 0.786, 1.180, 1.573, 1.966, 2.359$ by (Rao, Kantam, Rosaiah, & Prasad, 2012) along with the associated performance characteristics like operating characteristics, producer’s risk, scope for variability of σ etc. A typical portion of tables of (Rao, Kantam, Rosaiah, & Prasad, 2012) for type-II exponentiated log-logistic distribution are reproduced in the Table 1 for $\alpha = 2, \beta = 2$.

In the present investigation, inequality (2.4) can be considered in a different way. Let us fix n and let r be a natural number less than n , so that as soon as the r^{th} ($r=c+1$) failure is observed, the process is stopped and the lot is rejected. Given $\sigma = \sigma_0$, the probability of such a rejection should be as small as possible. That is

$$\sum_{i=r}^n \binom{n}{i} p^i (1-p)^{n-i} < \alpha^* \quad (2.5)$$

Specifying n as a multiple of r say kr ($k = 1, 2, \dots$), inequality (2.5) can be regarded as an inequality in a single unknown in terms of t/σ with known α, β .

With the choice of r, k, α^* inequality (2.5) can be solved for the earliest p say p_0 from which the value of t/σ_0 can be obtained by inverting the $G(t; \alpha, \beta, \sigma)$ given by (2.1). The specified population average in terms of σ_0 can be used here to get the value of t called the termination time. These are presented in Table 2 for various values of $n, r=1(1)10, \alpha = 2, \beta = 2$ at $\alpha^* = 0.05, 0.01$.



Table 1: Minimum sample size necessary to assert the average life to exceed specified average life σ_0 , with probability $(1 - \alpha^*)$ and the corresponding acceptance number c , using binomial probabilities for $\alpha = 2, \beta = 2$.

p^*	c	t/σ_0							
		0.315	0.472	0.629	0.786	1.180	1.573	1.966	2.359
0.75	0	8	4	3	2	1	1	1	1
0.75	1	15	8	5	4	3	2	2	2
0.75	2	22	11	7	6	4	3	3	3
0.75	3	29	15	10	8	5	5	4	4
0.75	4	36	18	12	9	7	6	5	5
0.75	5	42	22	14	11	8	7	6	6
0.75	6	49	25	17	13	9	8	8	7
0.75	7	55	28	19	15	10	9	9	8
0.75	8	62	32	21	16	12	10	10	9
0.75	9	68	35	23	18	13	11	11	10
0.75	10	75	38	26	20	14	13	12	11
0.90	0	13	6	4	3	2	1	1	1
0.90	1	22	11	7	5	3	3	2	2
0.90	2	30	15	10	7	5	4	4	3
0.90	3	37	19	12	9	6	5	5	4
0.90	4	45	23	15	11	8	6	6	6
0.90	5	52	26	17	13	9	8	7	7
0.90	6	59	30	20	15	10	9	8	8
0.90	7	66	33	22	17	12	10	9	9
0.90	8	73	37	24	19	13	11	10	10
0.90	9	80	41	27	20	14	12	11	11
0.90	10	87	44	29	22	16	13	12	12
0.95	0	16	8	5	4	2	2	1	1
0.95	1	26	13	8	6	4	3	3	2
0.95	2	35	17	11	8	5	4	4	4
0.95	3	43	21	14	10	7	6	5	5
0.95	4	51	25	16	12	8	7	6	6
0.95	5	59	29	19	14	10	8	7	7
0.95	6	66	33	22	16	11	9	8	8
0.95	7	74	37	24	18	12	10	10	9
0.95	8	81	41	27	20	14	12	11	10
0.95	9	88	44	29	22	15	13	12	11
0.95	10	96	48	31	24	16	14	13	12
0.99	0	25	12	7	5	3	2	2	2
0.99	1	36	18	11	8	5	4	3	3
0.99	2	46	22	14	10	7	5	5	4
0.99	3	55	27	17	13	8	6	6	5
0.99	4	64	31	20	15	10	8	7	6
0.99	5	73	36	23	17	11	9	8	8
0.99	6	81	40	26	19	13	10	9	9
0.99	7	89	44	28	21	14	11	10	10
0.99	8	97	48	31	23	15	13	11	11
0.99	9	105	52	34	25	17	14	13	12
0.99	10	112	56	36	27	18	15	14	13



Table 2. Life test termination time in units of scale parameter (t/σ_0) in generalized log- logistic distribution for $\alpha = 2, \beta = 2$.

r	$n = 2r$	$3r$	$4r$	$5r$	$6r$	$7r$	$8r$	$9r$	$10r$
$\alpha^* = 0.05$									
1	0.1138	0.0928	0.0804	0.0720	0.0658	0.0610	0.0567	0.0535	0.0511
2	0.2297	0.1817	0.1551	0.1376	0.1249	0.1152	0.1075	0.1013	0.0958
3	0.2945	0.2298	0.1950	0.1724	0.1563	0.1438	0.1341	0.1260	0.1194
4	0.3364	0.2603	0.2201	0.1943	0.1759	0.1619	0.1507	0.1416	0.1339
5	0.3662	0.2818	0.2377	0.2095	0.1895	0.1743	0.1622	0.1524	0.1442
6	0.3887	0.2979	0.2510	0.2209	0.1997	0.1835	0.1708	0.1605	0.1517
7	0.4066	0.3106	0.2613	0.2299	0.2077	0.1909	0.1776	0.1668	0.1576
8	0.4213	0.3209	0.2697	0.2372	0.2142	0.1968	0.1831	0.1719	0.1625
9	0.4334	0.3295	0.2766	0.2432	0.2195	0.2016	0.1875	0.1762	0.1665
10	0.4438	0.3368	0.2826	0.2483	0.2242	0.2059	0.1915	0.1797	0.1699
$\alpha^* = 0.01$									
1	0.0506	0.0413	0.0361	0.0324	0.0292	0.0274	0.0255	0.0245	0.0235
2	0.1473	0.1169	0.1000	0.0888	0.0807	0.0745	0.0695	0.0654	0.0618
3	0.2128	0.1668	0.1417	0.1256	0.1138	0.1048	0.0977	0.0920	0.0871
4	0.2581	0.2006	0.1700	0.1503	0.1360	0.1254	0.1167	0.1096	0.1038
5	0.2915	0.2255	0.1906	0.1683	0.1523	0.1400	0.1305	0.1226	0.1161
6	0.3174	0.2445	0.2063	0.1820	0.1645	0.1514	0.1410	0.1323	0.1251
7	0.3383	0.2597	0.2190	0.1930	0.1743	0.1603	0.1492	0.1402	0.1327
8	0.3555	0.2723	0.2293	0.2019	0.1825	0.1678	0.1561	0.1465	0.1385
9	0.3700	0.2828	0.2380	0.2095	0.1892	0.1738	0.1619	0.1519	0.1436
10	0.3825	0.2918	0.2454	0.2159	0.1950	0.1791	0.1666	0.1564	0.1480

Table 3: Proportions of life test termination time for sampling plans of (Rao, Kantam, Rosaiah, & Prasad, 2012) and the present sampling plans with producer's risk $\alpha^* = 0.05$.

r	n	$2r$	$3r$	$4r$	$5r$	$6r$	$7r$	$8r$	$9r$	$10r$
1		0.1138 1.180		0.0804 0.786	0.0720 0.629			0.0567 0.472		
2		0.2297 1.180	0.1817 0.786	0.1551 0.629						
5					0.2095 0.472					
8			0.3209 0.629							
9			0.3295 0.629						0.1762 0.315	



Table 4: Proportions of life test termination time for sampling plans of (Rao, Kantam, Rosaiah, & Prasad, 2012) and the present sampling plans with producer’s risk $\alpha^* = 0.01$.

n	2r	3r	4r	5r	6r	7r	8r	9r	10r
r									
1	0.0506 1.573	0.0413 1.180		0.0324 0.786		0.0274 0.629			
2	0.1473 1.573		0.1000 0.786					0.0654 0.472	
4	0.2581 1.180								
5	0.2915 1.180	0.2255 0.786	0.1906 0.629						
6					0.1645 0.472				

2.1. Comparative Study

In order to compare the present sampling plan with that of (Rao, Kantam, Rosaiah, & Prasad, 2012), the entries common for both the approaches are presented for $\alpha = 2, \beta = 2; \alpha^* = 0.05, 0.01$ in Tables 3 and 4. The entries given in the first row are corresponding to present test plan and those given in the second row are obtained by (Rao, Kantam, Rosaiah, & Prasad, 2012). All the entries in Tables 3 and 4 show that for a given n, r ($r = c+1$), the values of t/σ_0 - the scaled termination time is uniformly smaller for the present reliability test plans than those of (Rao, Kantam, Rosaiah, & Prasad, 2012), resulting in savings in experimental time.

3. OPERATING CHARACTERISTIC FUNCTION

If the true but unknown life of the product deviates from the specified life of the product it should result in a considerable change in the probability of acceptance of the lot based on the sampling plan. Hence the probability of acceptance can be regarded as a function of the deviation of specified average from the true average. This function is called operating characteristic (OC) function of the sampling plan. Specifically if $G(t; \alpha, \beta, \sigma_0)$ is the cumulative distribution function of the life time random variable of the product, σ_0 corresponds to specified life, we can write

$$G(t/\sigma) = G\left[\left(t/\sigma_0\right) \cdot \left(\sigma_0/\sigma\right)\right] \tag{3.1}$$

where σ corresponds to true but unknown average life. The ratio σ_0/σ in the R.H.S. of above equation can be taken as a measure of changes between true and specified lives. For instance $(\sigma_0/\sigma) < 1$ implies true mean life is more than the declared life leading to more acceptance probability or less failure risk. Similarly σ_0/σ more than 1 implies less acceptance probability or more failure risk. Hence giving a set of hypothetical values say $\sigma_0/\sigma = 0.1(0.1)1.9$, we can have the corresponding acceptance probability for the given sampling plan. The graph between σ_0/σ and probabilities of acceptance given by equation (3.1) for a sampling plan forms the OC. curve of that plan. Here we have selected some plans and OC. values of these plans are given in Table 5 and the corresponding OC. curves are also drawn in Figure 1.

3.1. Example

Consider the following ordered failure times of the release of software given in terms of hours from the starting of the execution of the software denoting the times at which the failure of the software is experienced (Wood, 1996). This data can be regarded as an ordered sample of size $n = 8$ with observations.

$$\{t_i : i = 1, 2, \dots, 8\} = \left\{ \begin{array}{l} 519, 968, 1430, 1893, 2490, \\ 3058, 3625, 4422, 5218 \end{array} \right\}$$

The confidence level of the decision processes is assured by the sampling plan, only if the lifetimes follow generalized log-logistic distribution. We have verified this for the above sample data by Q-Q plot with $\alpha = 2, \beta = 2$.



Case I:

Let the required average lifetime be 1000 hours and the testing time be $t=472$ hours, this leads to ratio of $t/\sigma_0=0.472$ with a corresponding sample size $n=8$ and an acceptance number $c=0$, which are obtained from Table 1 for $1-\alpha^*=0.95$. Therefore, the sampling plan for the above sample data is ($n=8, c=0, t/\sigma_0=0.472$). Based on the observations, we have to decide whether to accept the product or reject it. We accept the product only if the number of failures before 472 hours is less than or equal to $c=0$. In the above sample of 8 no failure occurred at before 519hours and hence before $t=472$ hours also. Therefore we accept the product using the sampling plan constructed by (Rao, Kantam, Rosaiah, & Prasad, 2012).

Case II:

From Table 2, the entry against $r=1$ ($r=c+1$) under the column 8r is 0.0567. Since the acceptable mean life is given to be 1000 hours for generalized log-logistic distribution, if the termination time is given by ' t_0 ' the table value says that

$$t/\sigma_0 = 0.0567 \text{ that is } t_0 = 0.0567 \times 1000 = 56.7$$

hours = 57hours (approx.).

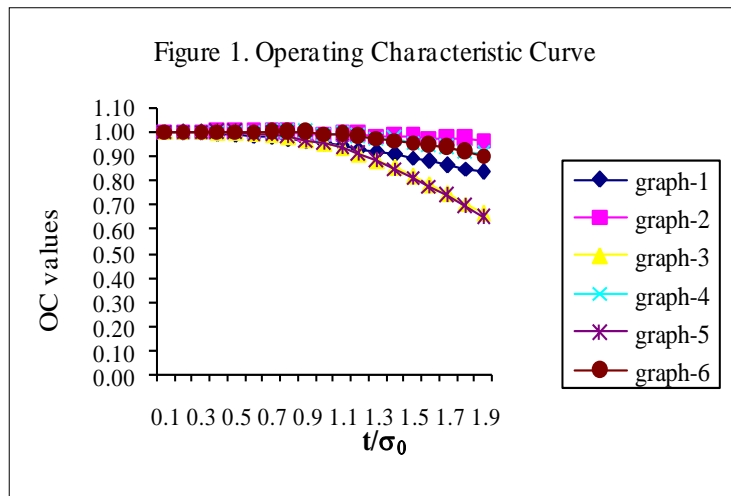
Using the present sampling plan, this test plan will be implemented as follows: Select 8 items from the submitted lot and put them to test. If the first failure is realized before 57th hour of the test, reject the lot otherwise accept the lot, in either case terminating the experiment as soon as the first failure is reached or 57th hour of the test time is reached whichever is earlier. In the case of acceptance, the assurance is that the average life of the submitted products is at least 1000 hours.

In this approach, we see that in the sample of 8 failures there is no failure before 57th hour, therefore we accept the product.

In both of these approaches the sample size, acceptance number (termination number), the risk probability and the final decision about the lot are the same. But the decision on the first approach can be reached at the 472nd hour and that in the second approach reached at the 57th hour. Thus the present sampling plan requiring a less waiting time and also minimum experimental cost is preferable to that of first approach [(Rao, Kantam, Rosaiah, & Prasad, 2012)].

Table 5: Operating characteristic (O.C) values of sampling plans ($n, r, t/\sigma_0$) for $\alpha = 2, \beta = 2$.

	n=2, r=1		n=4, r=2		n=8, r=2	
	t/σ_0		t/σ_0		t/σ_0	
	0.1138	0.0506	0.2297	0.1473	0.1551	0.1000
σ_0/σ	$1-\alpha^*=0.95$	$1-\alpha^*=0.99$	$1-\alpha^*=0.95$	$1-\alpha^*=0.99$	$1-\alpha^*=0.95$	$1-\alpha^*$
0.1	0.9995	0.9999	1.0000	1.0000	1.0000	1.0000
0.2	0.9979	0.9996	0.9999	1.0000	0.9999	1.0000
0.3	0.9954	0.9991	0.9995	0.9999	0.9995	0.9999
0.4	0.9918	0.9984	0.9984	0.9997	0.9984	0.9997
0.5	0.9872	0.9974	0.9961	0.9993	0.9962	0.9993
0.6	0.9816	0.9963	0.9922	0.9986	0.9924	0.9986
0.7	0.9750	0.9950	0.9861	0.9974	0.9863	0.9975
0.8	0.9675	0.9935	0.9773	0.9957	0.9775	0.9957
0.9	0.9591	0.9917	0.9653	0.9933	0.9655	0.9933
1.0	0.9498	0.9898	0.9499	0.9900	0.9499	0.9900
1.1	0.9397	0.9877	0.9307	0.9857	0.9304	0.9856
1.2	0.9288	0.9854	0.9078	0.9803	0.9069	0.9801
1.3	0.9170	0.9829	0.8812	0.9736	0.8794	0.9733
1.4	0.9046	0.9802	0.8510	0.9656	0.8479	0.9652
1.5	0.8915	0.9773	0.8177	0.9562	0.8129	0.9555
1.6	0.8777	0.9742	0.7816	0.9453	0.7747	0.9442
1.7	0.8633	0.9709	0.7431	0.9328	0.7339	0.9313
1.8	0.8484	0.9675	0.7030	0.9188	0.6909	0.9167
1.9	0.8330	0.9639	0.6616	0.9032	0.6464	0.9004



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